*δ-Functions in Topological Spaces

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Abstract:- In this paper, we shall introduce and study on some new type of generalized functions called *δ-continuous, *δ-irresolute, functions via the concept of *δ –sets. The purpose of this work is to explore properties and characterizations of these functions. Using the concept of *δ -open sets, we also introduce *δ –open mappings and investigate certain characterizations of this type of functions.

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1. Introduction

In 1970, Levine [4] introduced and studied generalized closed sets in topological spaces. Dunham [3] introduced the concept of generalized closure operator, using Levine's generalized closed sets and defined a new topology τ^* and studied its properties. Velico [9] introduced the notion of δ-closure and δ -interior operations.

Noiri [7], Balachandran et al [1], Dontchev.J and Ganster [2] introduced δ-continuity, generalized continuous function and δ-generalised continuous δ-generalised irresolute functions respectively. Sundaram et al [8] introduced semi-generalised continuity in topological spaces. Pious Missier et al [7] introduced $regular^*$-continuous functions.

The purpose of this present paper, we shall introduce and study on some new type of generalized functions called *δ-continuous, *δ-irresolute, functions via the concept of *δ –sets. The purpose of this work is to explore properties and characterizations of these functions. Using the concept of *δ -open sets, we also introduce *δ –open mappings and investigate certain characterizations of this type of functions. Throughout the present paper (X,τ) (Simply X) always mean topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, $cl(A)$ and $int(A)$ denote the closure and the interior of A respectively. Now we recall some of the basic definitions and results in topology.

2. Preliminaries

Definition 2.1.[5] The *δ-interior of a subset A of X is the union of all $regular^*$-open sets of X contained in A and is denoted by $int_{*_\delta}(A)$.

Definition 2.2. [5] A subset A of a topological space (X,τ) is called *δ-open if $A = int_{*_\delta}(A)$ i.e., a set is *δ-open if it is the union of $regular^*$-open sets. The complement of a *δ-open is called *δ-closed set in X.

Notations 2.3.[5] The set of all *δ-open sets in (X,τ) is denoted by *δ$O(X)$.

Definition 2.4. [6] A function $f:(X,\tau)\to(Y,\sigma)$ is said to be δ-continuous if $f^{-1}(V)$ is δ-open in (X,τ) for every δ-open set V of (Y,σ).

Definition 2.5. [7] A function $f:(X,\tau)\to(Y,\sigma)$ is said to be $regular^*$-continuous if $f^{-1}(V)$ is $regular^*$-open in (X,τ) for every open set V of in Y.

Theorem 2.6. [5] For a topological space (X,τ), the family of all *δ-open sets of X forms a topology, denoted by τ_{*_δ}, for X.

3. *δ-continuous functions

The purpose of this section is to introduce *δ-continuous function and to investigate further properties and characterization of this function.

Definition 3.1. A function $f:(X,\tau) \to (Y,\sigma)$ is said to be *δ-continuous if $f^{-1}(V)$ is *δ-closed in (X,τ), for every closed set in (Y,σ).

Definition 3.2. Let (X,τ) be a topological space. Let $x \in X$ and $N \subseteq X$. We say that N is a *δ-neighborhood of x if there exists a *δ-open set M of X such that $x \subset M \subseteq N$.

Theorem 3.3. Let $f:(X,\tau) \to (Y,\sigma)$ be a function. Then the following are equivalent.

(i) f is *δ-continuous;

(ii) The inverse image of each closed set in Y is a *δ-closed in X;

(iii) $cl_{*\delta}[f^{-1}(V)] \subseteq f^{-1}[cl(V)]$, for every $V \subseteq Y$

(iv) $f(cl_{*\delta}(U)) \subseteq cl(f(U))$, for every $U \subseteq X$

(v) For any point $x \in X$ and any open set V of Y containing $f(x)$, there exists

$U \in \tau_{*\delta}$ such that $x \in U$ and $f(U) \subseteq V$

(vi) $Bd_{*\delta}[f^{-1}(V)] \subseteq f^{-1}[Bd(V)]$, for every $V \subseteq Y$

(vii) $f\left(D_{*\delta}(U)\right) \subseteq cl[f(U)]$, for every $U \subseteq X$

(viii) $f^{-1}[int(V)] \subseteq int_{*\delta}[f^{-1}(V)]$, for every $V \subseteq Y$.

Proof: $(i) \Longrightarrow (ii)$ Let F be a closed set in Y. Since f is *δ-continuous, $f^{-1}(Y \quad F) = X \quad f^{-1}(F)$ is *δ-open. Hence $f^{-1}(F)$ is *δ-closed in X.

$(ii) \longrightarrow (iii)$ Since $cl(V)$ is a closed for every $V \subseteq Y$, $f^{-1}[cl(V)]$ is *δ-closed. Therefore $f^{-1}[cl(V)] = cl_{*\delta}[f^{-1}(cl(V))] \supseteq cl_{*\delta}[f^{-1}(V)]$.

$(iii) \Longrightarrow (iv)$ Let $U \subseteq X$ and $f(U) = V$. Given $f^{-1}[cl(V)] \supseteq cl_{*\delta}[f^{-1}(V)]$. Thus $f^{-1}[cl(f(U))] \supseteq cl_{*\delta}[f^{-1}(f(U)]$. Hence $cl[f(U)] \supseteq f[cl_{*\delta}(U)]$.

$(iv) \longrightarrow (ii)$ Let W be a closed set in Y and $U = f^{-1}(W)$, then $f[cl_{*\delta}(U)] \subseteq cl(f(U)) - cl(f(f^{-1}(W)) \subseteq cl(W) - W$. Here $f[cl_{*\delta}(U)] \subseteq f(U)$. Thus $cl_{*\delta}(U) \subseteq f^{-1}[cl(f(U))] = f^{-1}[cl(W)] = f^{-1}(W) = U$. So U is *δ-closed.

$(ii) \Longrightarrow (i)$ Let $V \subseteq Y$ be an open set, then $Y - V$ is closed. Then $f^{-1}(Y - V) = X - f^{-1}(V)$ is *δ-closed in X. Hence $f^{-1}(V)$ is δ-open in X.

$(i) \Longrightarrow (v)$ Let $f:(X,\tau) \to (Y,\sigma)$ be *δ-continuous. For any $x \in X$ and any open set V of Y containing $f(x)$, $U = f^{-1}(V) \in \tau_{*\delta}$, and $f(U) - f[f^{-1}(V)] \subseteq V$.

$(v) \Longrightarrow (i)$ Let V be any set in Y. Let $x \in f^{-1}(V)$. Then $f(x) \in V$ and there exists $U \in \tau_{*\delta}$ such that $x \in U$ and $f(x) \in f(U) \subseteq V$. Then $x \in U \subseteq f^{-1}[f(U)] \subseteq f^{-1}(V)$. It shows that $f^{-1}(V)$ is a *δ-neighborhood of each of its points. Therefore $f^{-1}(V) \in \tau_{*\delta}$.

$(vi) \Longrightarrow (viii)$ Let $V \subseteq Y$. Then by hypothesis, $Bd_{*\delta}[f^{-1}(V)] \subseteq f^{-1}[Bd(V)]$ which implies $f^{-1}(V) - int_{*\delta}[f^{-1}(V)] \subseteq f^{-1}(V - int(V)) = f^{-1}(V) - f^{-1}(int(V))$, implies that $f^{-1}(int(V)) \subseteq int_{*\delta}[f^{-1}(V)]$.

$(viii) \Rightarrow (vi)$ Let $V \subseteq Y$. Then by hypothesis, $f^{-1}[int(V)] \subseteq int_{*_\delta}[f^{-1}(V)]$ which implies $f^{-1}(V) - int_{*_\delta}[f^{-1}(V)] \subseteq f^{-1}(V) - f^{-1}[int(V)] - f^{-1}[V - int(V)] \rightarrow Bd_{*_\delta}[f^{-1}(V)] \subseteq f^{-1}[Bd(V)]$.

$(i) \Rightarrow (vii)$ It is obvious, since f is *δ-continuous and by (iv), $f[cl_{*_\delta}(U)] \subseteq cl[f(U)]$ for each $U \subseteq X$. So $f[D_{*_\delta}(U)] \subseteq cl(f(U))$.

$(vii) \rightarrow (i)$ Let U be an open set in Y, $V = Y - U$ and $f^{-1}(V) = W$. Then by hypothesis, $f[D_{*_\delta}(W)] \subseteq cl(f(W))$. Thus $f[D_{*_\delta}(f^{-1}(V)] \subseteq cl[f(f^{-1}(V))] \subseteq cl(V) = V$. Then $D_{*_\delta}(f^{-1}(V)] \subseteq f^{-1}(V)$ and $f^{-1}(V)$ is *δ-closed. Hence f is *δ-continuous.

$(i) \Rightarrow (viii)$ Let $V \subseteq Y$. Then $f^{-1}(int(V))$ is *δ-open in X. Thus $f^{-1}(int(V)) = int_{*_\delta}\left(f^{-1}(int(V)) \subseteq int_{*_\delta}(f^{-1}(V)\right)$. Therefore, $f^{-1}(int(V)) \subseteq int_{*_\delta}(f^{-1}(V)$.

$(viii) \Rightarrow (i)$ Let $V \subset Y$ be an open set. Then $f^{-1}(V) = f^{-1}(int(V)) \subseteq int_{*_\delta}(f^{-1}(V))$ Hence $f^{-1}(V)$ is *δ-open. Hence f is *δ-continuous.

Theorem 3.4. Every $regular^*$- continuous function is *δ-continuous.

Proof: It is true that every $regular^*$-open set is *δ-open.

Remark 3.5. The converse of the above theorem is not true in general as shown in the following example.

Example3.6. Let $X = \{a, b, c\}$; $Y = \{p, q, r\}$. Let $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{q\}, \{r\}, \{q, r\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = r, f(b) = p, f(c) = q$. Then f is not $regular^*$-continuous function because the set $\{q, r\}$ is open in Y, but $f^{-1}(q, r) - \{a, c\}$ is not $regular^*$-open in X. However f is *δ-continuous.

Theorem 3.7. Every *δ-continuous is continuous.

Proof: Follows from the definitions.

Remark 3.8. The converse of Theorem 3.7, is not true in general as shown in the following example.

Example 3.9. Let $X = \{a, b, c\}$; $Y = \{p, q, r\}$ with the topologies $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{q, r\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = p, f(b) = r, f(c) = q$. Then f is not *δ--continuous function because the set $\{q, r\}$ is open in Y, but $f^{-1}(q, r) = \{b, c\}$ is not *δ-open in X. However f is continuous function.

Theorem 3.10. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two functions. Then

(i) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is *δ-continuous if g is continuous and f is *δ-continuous.

(ii) $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is δ-continuous if g is *δ-continuous and f is δ-continuous.

Proof: (i) Let F be open set in (Z, η). Then $g^{-1}(F)$ is open in (Y, σ). Since f is *δ-continuous, $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is *δ-open set in (X, τ) and so $g \circ f$ is *δ-continuous function.

(ii) Let F be open set in (Z, η). Then $g^{-1}(F)$ is *δ-open in (Y, σ). Since every *δ-open is open and since f is δ-continuous, $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is δ-open set of (X, τ) and so $g \circ f$ is δ-continuous function.

Lemma 3.11. Let $A \subseteq Y \subseteq X, Y$ is *δ-open in X and A is *δ-open in (Y, σ). Then A is *δ-open in X.

Proof: Since A is *δ-open in Y, there exists a *δ-open set $U \subseteq X$ such that $A = Y \cap U$. Hence A being the intersection of two *δ-open sets in X is *δ-open in X.

Theorem 3.12. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping and $\{U_i : i \in I\}$ be a cover of X such that $U_i \in \tau_{*_\delta}$ for each $i \in I$. Suppose that $f/U_i : U_i \rightarrow Y$ is *δ-continuous for each $i \in I$. Then f is *δ-continuous.

Proof: Let $V \subseteq Y$ be an open set, then $(f/U_i)^{-1}(V)$ is *δ-open in U_i for each $i \in I$. Since U_i is *δ-open in X, for each $i \in I$ and by Lemma 2.5, $(f/U_i)^{-1}(V)$ is *δ-open in X for each $i \in I$. But $f^{-1}(V) = \cup\{(f/U_i)^{-1}(V): i \in I\}$, then $f^{-1}(V) \in \tau_{*_\delta}$ because τ_{*_δ} is a topology on X. This implies that f is *δ-continuous.

4. *δ-irresolute functions

In this section we introduce *δ-irresolute functions using *δ-open sets. We investigate some properties and characterization of such functions.

Definition 4.1. Let (X,τ) and (Y,σ) be topological spaces. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be *δ-irresolute if the inverse image of each *δ-open set of Y is a *δ-open set in X .

Theorem 4.2. Let $f:(X,\tau)\to(Y,\sigma)$ be a function. Then the following are equivalent.

(i) f is *δ-irresolute

(ii) The inverse image of each *δ-closed set in Y is a *δ-closed set in X.

(iii) $cl_{*\delta}[f^{-1}(V)]\subseteq f^{-1}[cl_{*\delta}(V)]$ for every $V\subseteq Y$.

(iv) $f[cl_{*\delta}(U)]\subseteq cl_{*\delta}[f(U)]$ for every $U\subseteq X$.

(v) $f^{-1}[int_{*\delta}(V)]\subseteq int_{*\delta}(f^{-1}(V))$ for every $V\subseteq Y$.

Theorem 4.3. A function $f:(X,\tau)\to(Y,\sigma)$ is *δ-irresolute iff for each point x in X and each *δ-open set U in Y with $f(x)\in U$, there is a *δ-open set V in X such that $x\in V,\ f(V)\subseteq U$.

Proof: Necessity: Let $x\in X$ and let U be an *δ-open set in Y such that $f(x)\subset U$. Let $V=f^{-1}(U)$. Since f is *δ-irresolute, V is *δ-open in X. Also $x\in f^{-1}(U)=V$ as $f(x)\in U$. Hence we have $f(V)=f[f^{-1}(U)]\subseteq U$.

Sufficiency: Let U be an *δ-open set in Y and let $V=f^{-1}(U)$ Let $x\in V$ then $f(x)\in U$. Then by hypothesis, there exists $V_x\in\tau_{*\delta}$ such that $x\in V_x$ and $f(V_x)\subseteq U$. Then $V_x\subseteq f^{-1}[f(V_x)]\subseteq f^{-1}(U)=V$. Thus $V=\cup\{V_x : x\in V\}$. It follows that V is *δ-open in X. Hence f is *δ-irresolute.

Theorem 4.4. A function $f:(X,\tau)\to(Y,\sigma)$ is *δ-irresolute iff for each x in X and the inverse image of every *δ-neighborhood of $f(x)$ is a *δ- neighborhood of x.

Proof : Necessity: Let $x\in X$ and let B be an *δ- neighborhood of $f(x)$. Then there exists $U\in\sigma_{*\delta}$ such that $f(x)\in U\subseteq B$. This implies that $x\in f^{-1}(U)\subseteq f^{-1}(B)$. Since f is *δ-irresolute, $f^{-1}(U)\in\tau_{*\delta}$. Hence $f^{-1}(B)$ is a *δ-neighborhood of x.

Sufficiency: Let $B\in\sigma_{*\delta}$, $A=f^{-1}(B)$ and let $x\in A$. Then $f(x)\in B$. Since B is *δ- open set, B is a *δ- neighborhood of $f(x)$, so by hypothesis $A=f^{-1}(B)$ is a *δ-neighborhood of x. Hence by definition, there exists $A_x\in\tau_{*\delta}$ such that $x\subset A_x\subseteq A$. Thus A$=\cup\{A_x : x\subset A\}$. It follows that A is a *δ-open set in X. Therefore f is *δ-irresolute.

Theorem 4.5. A function $f:(X,\tau)\to(Y,\sigma)$ is *δ-irresolute iff for each x in X and each *δ- neighborhood U of $f(x)$, there is a *δ- neighborhood V of x such that $f(V)\subset U$.

Proof: Necessity: Let $x\in X$ and let U be a *δ- neighborhood of $f(x)$. Then there exists $O_{f(x)}\in\sigma_{*\delta}$ such that $f(x)\in O_{f(x)}\subseteq U$. Then $x\in f^{-1}(O_{f(x)})\subseteq f^{-1}(U)$. By hypothesis, $f^{-1}(O_{f(x)})\in\tau_{*\delta}$. Let $V=f^{-1}(U)$. Then it follows that V is a *δ- neighborhood of x and $f(V)=f[f^{-1}(U)]\subseteq U$.

Sufficiency: Let $B\in\sigma_{*\delta}$. Let $O=f^{-1}(B)$. Let $x\in O$. Then $f(x)\in B$. Thus B is a *δ- neighborhood of $f(x)$. By hypothesis, there exists *δ- neighborhood V_x of x such that $f(V_x)\subseteq B$. Hence it follows that $x\in V_x\subseteq f^{-1}[f(V_x)]\subseteq f^{-1}(B)=O$. Since V_x is a *δ- neighborhood of x, there exists $O_{f(x)}\in\tau_{*\delta}$ such that $x\in O_x\subseteq V_x$. Then $x\in O_x\subseteq O$, $O_x\in\tau_{*\delta}$. Thus O$=\cup\{O_x : x\in O\}$. It follows that O is δ-open in X. Therefore f is *δ-irresolute.

Theorem 4.6. A function $f:(X,\tau)\to(Y,\sigma)$ be a one-to-one function. Then f is *δ-irresolute iff $f[D_{*\delta}(A)]\subseteq D_{*\delta}[f(A)]$, for all $A\subseteq X$.

Proof: Necessity: Let f be a *δ-irresolute. Let $A \subseteq X, a_0 \in D_{*_\delta}(A)$ and V be a *δ- neighborhood of $f(a_0)$. Since f is *δ-irresolute, by Theorem 4.5, there exists a f is *δ- neighborhood of U of a_0 such that $f(U) \subseteq V$. But $a_0 \in D_{*_\delta}(A)$ then there exists an element $a \in U \cap A$ such that $a \neq a_0$ hence $f(a) \in f(A)$ and since f is 1-1 function, $f(a) \neq f(a_0)$. Thus every *δ- neighborhood of V of $f(a_0)$ contains an element of $f(A)$ different from $f(a_0)$, consequently $f(a_0) \in D_{*_\delta}[f(A)]$. Therefore $f[D_{*_\delta}(A)] \subseteq D_{*_\delta}[f(A)]$.

Sufficiency: Suppose that f is not a *δ-irresolute. Then by Theorem 4.5, there exists $a_0 \in X$ and a *δ- neighborhood of V of $f(a_0)$ such that every *δ- neighborhood U of a_0 contains at least one element $a \in U$ for which $f(a) \notin V$. Let $A = \{a \subset X: f(a) \not\subset V\}$. Then $a_0 \notin A$ since $f(a) \subset V, f(a) \neq f(a_0)$, also $f(a_0) \notin D_{*_\delta}[f(A)]$ since $f(A) \cap [V \quad \{f(a_0)\}] = \emptyset$. It follows that $f(a_0) \in f[D_{*_\delta}(A)] - [f(A) \cup D_{*_\delta}(f(A))] \neq \emptyset$, this contradicts gives the proof.

5. *δ-open mappings

The purpose of this section is to introduce *δ-open mappings and investigate certain properties & characterization of this type of functions.

Definition 5.1. Let (X, τ) and (Y, σ) be topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be *δ-open if for every open set U in X, $f(U)$ is a *δ-open set in Y.

Theorem 5.2. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is a *δ-open iff for each $x \in X$, and $U \in \tau$ such that $x \in U$, there exists a *δ-open set $W \subseteq Y$ containing $f(x)$ such that $W \subseteq f(U)$.

Proof: Follows from the definitions.

Theorem 5.3. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a *δ-open. If $W \subseteq Y$ and $F \subseteq X$ is a closed set containing $f^{-1}(W)$, then there exists a *δ-closed $H \subset Y$ containing W such that $f^{-1}(H) \subseteq F$.

Proof: Let $H = Y - f(X - F)$. Since $f^{-1}(W) \subseteq F, f(X - F) \subseteq Y - W$. Also since f is *δ-open, H is *δ-closed and $f^{-1}(H) - X - f^{-1}[f(X - F)] \subseteq X - (X - F) - F$.

Theorem 5.4. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is *δ-open iff $f(int(A)) \subseteq int_{*_\delta}[f(A)]$, for all $A \subseteq X$.

Proof: Necessity: Let $A \subseteq X$. Let $x \in int(A)$. Then there exists $U_x \in \tau$ such that $x \in U_x \subseteq A$. Hence $f(x) \in f(U_x) \subseteq f(A)$ and by hypothesis, $f(U_x) \in \sigma_{*_\delta}$. Thus $f(x) \in int_{*_\delta}[f(A)]$. Therefore $f[int(A)] \subset int_{*_\delta}[f(A)]$.

Sufficiency: Let U be any open set in X. Then by hypothesis, $f[int(U)] \subseteq int_{*_\delta}[f(U)]$. Since $int(U) = U$ as U is open. Also $int_{*_\delta}[f(U)] \subseteq f(U)$. Hence $f(U) = int_{*_\delta}[f(U)]$. Therefore $f(U)$ is *δ-open in Y. So f is *δ-open.

Theorem 5.5. The mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is *δ-open iff $int[f^{-1}(B)] \subseteq f^{-1}(int_{*_\delta}(B)$ for all B$\subseteq Y$.

Proof: Necessity: Let B$\subseteq Y$. Since $int[(f)^{-1}(B)]$ is open in X and f *δ-open, $f[int(f)^{-1}(B)]$ is *δ-open in Y. Also $f[int([(f)^{-1}(B))] \subseteq f[f^{-1}(B)] \subseteq B$. Thus $f[int(f^{-1}(B))] \subset int_{*_\delta}(B)$. Therefore $int[f^{-1}(B)] \subset f^{-1}(int_{*_\delta}(B)$.

Sufficiency: Let $A \subseteq X$. Then $f(A) \subset Y$. Then by hypothesis, $int(A) \subset int[f^{-1}(f(A))] \subset f^{-1}(int_{*_\delta}(f(A))$. Thus $f[int(A)] \subseteq int_{*_\delta}(f(A))$, for all $A \subseteq X$. Hence by the above Theorem, f is *δ-open.

Theorem 5.6. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then a necessary and sufficient condition for f to be *δ-open is that $f^{-1}(cl_{*_\delta}(B)) \subset cl[f^{-1}(B)]$ for every subset B of Y.

Proof: Necessity: Let f be an *δ-open. Let B$\subseteq Y$ and let $x \in f^{-1}(cl_{*_\delta}(B))$. Then $f(x) \in cl_{*_\delta}(B))$. Let $U \in \tau$ such that $x \in U$. Since f is *δ-open, then $f(U)$ is a *δ-open set in Y. Hence $B \cap f(U) \neq \emptyset$. Then $U \cap f^{-1}(B) \neq \emptyset$. Therefore $x \in cl[f^{-1}(B)]$. Hence $f^{-1}[cl_{*_\delta}(B)] \subset cl[f^{-1}(B)]$.

Sufficiency: Let $B \subseteq Y$. Then $(Y - B) \subset Y$. By hypothesis, $f^{-1}[cl_{*_\delta}(Y - B) \subseteq cl[f^{-1}(Y - B)]$. Then $X - cl[f^{-1}(Y - B)] \subseteq X - f^{-1}[cl_{*_\delta}(Y - B)]$. Hence $X - cl[X - f^{-1}(B)] \subseteq f^{-1}[Y - cl_{*_\delta}(Y - B)]$. Hence $int[f^{-1}(B)] \subseteq f^{-1}[int_{*_\delta}(B)]$. Now from Theorem 5.5, it follows that f is *δ-open.

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